Documentation on octree and quadrant connectivity

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1 Mappings between octrees and physical space

See Figure 1 for an illustration.

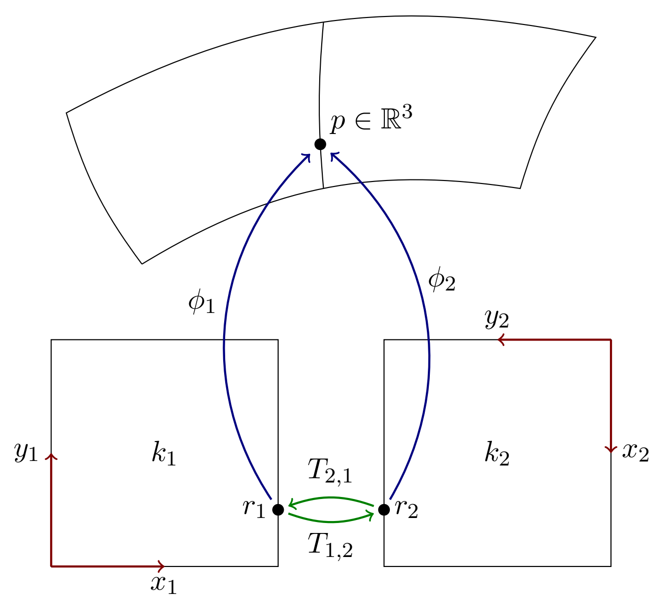

Figure 1: Two octrees k_1 and k_2 and their individual octree coordinate systems (red). Quadrant coordinates at subdivision level $\ell \geq 0$ (not shown) are discrete numbers that are integer multiples of $2^{-\ell}$. In this example the octrees connect through a common face, which defines coordinate transformations $T_{1,2} = T_{2,1}^{-1}$ across this face. The octree points $r_1 = (1, \frac{1}{4})$ and $r_2 = (\frac{3}{4}, 1)$ are identified: $T_{i,j}(r_i) = r_j$ (green). This identification is specified solely based on the connectivity relation between the two octrees. The functions T_i can be implemented in integer arithmetic, entirely without using physical coordinates. To map the octrees to physical space, geometry transformations ϕ_1, ϕ_2 (blue) are introduced that satisfy the compatibility condition $p = \phi_1(r_1) = \phi_2(r_2)$. This offers the alternate identification criterion $r_j = (\phi_j^{-1} \circ \phi_i)(r_i)$. However, this approach is not recommended since it is much more expensive and, more importantly, depends on the floating-point functions ϕ_i that suffer from roundoff error and make it difficult to determine uniqueness.